Calculation of double, successive, and radial limits

Calculate, if they exist, the double limit, the successive limits, and the radial limit of the following functions at the indicated points.

$$\lim_{(x,y)\to(0,0)} y \sin\left(\frac{1}{x}\right)$$

Solution

First, we calculate the double limit

$$\lim_{(x,y)\to(0,0)} y\sin\left(\frac{1}{x}\right)$$

This is a product of two functions, one of which tends to 0 and the other is bounded since $-1 < \sin(x) < 1$, thus $-1 < \sin(1/x) < 1$. Therefore, by the theorem of a bounded function multiplied by an infinitesimal, we can affirm that

$$\lim_{(x,y)\to(0,0)} y\sin\left(\frac{1}{x}\right) = 0$$

The successive limits:

$$L_1 = \lim_{(y)\to 0}[\lim_{(x)\to 0} y\sin\left(\frac{1}{x}\right)]$$

L_1 does not exist.

$$L_2 = \lim_{(x)\to 0}[\lim_{(y)\to 0} y\sin\left(\frac{1}{x}\right)] = \lim_{(x)\to 0} 0 = 0$$

Finally, the radial limit:

$$y - y_0 = m(x - x_0)$$

$$y = mx$$

$$L_r = \lim_{x\to 0} mx\sin(1/x) = 0$$

Which tends to 0 by the theorem of an infinitesimal multiplied by a bounded function.